

Accounting for correlated observation error in variational ocean data assimilation: application to wide-swath altimeter data

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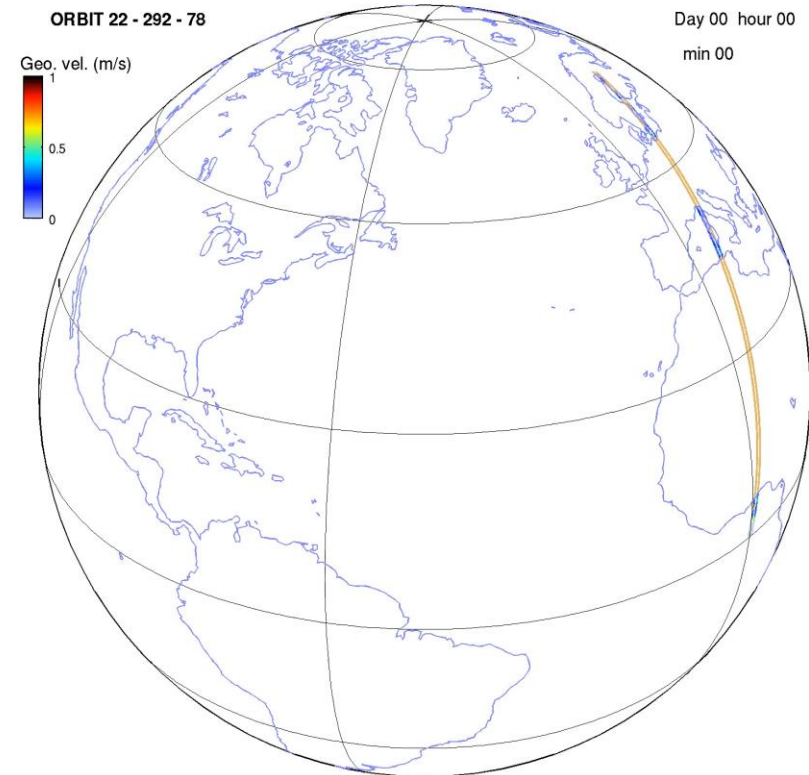
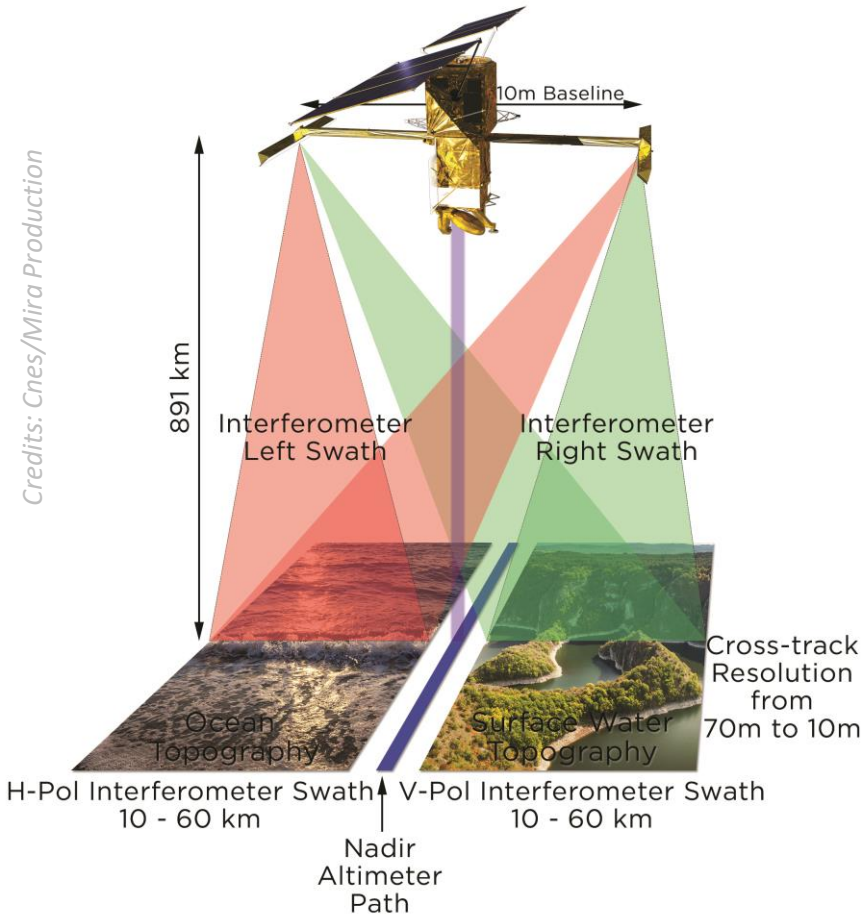
1, CERFACS, Toulouse, France

2, Météo-France, Toulouse, France

3, Polytechnique Montréal, Montréal, Canada

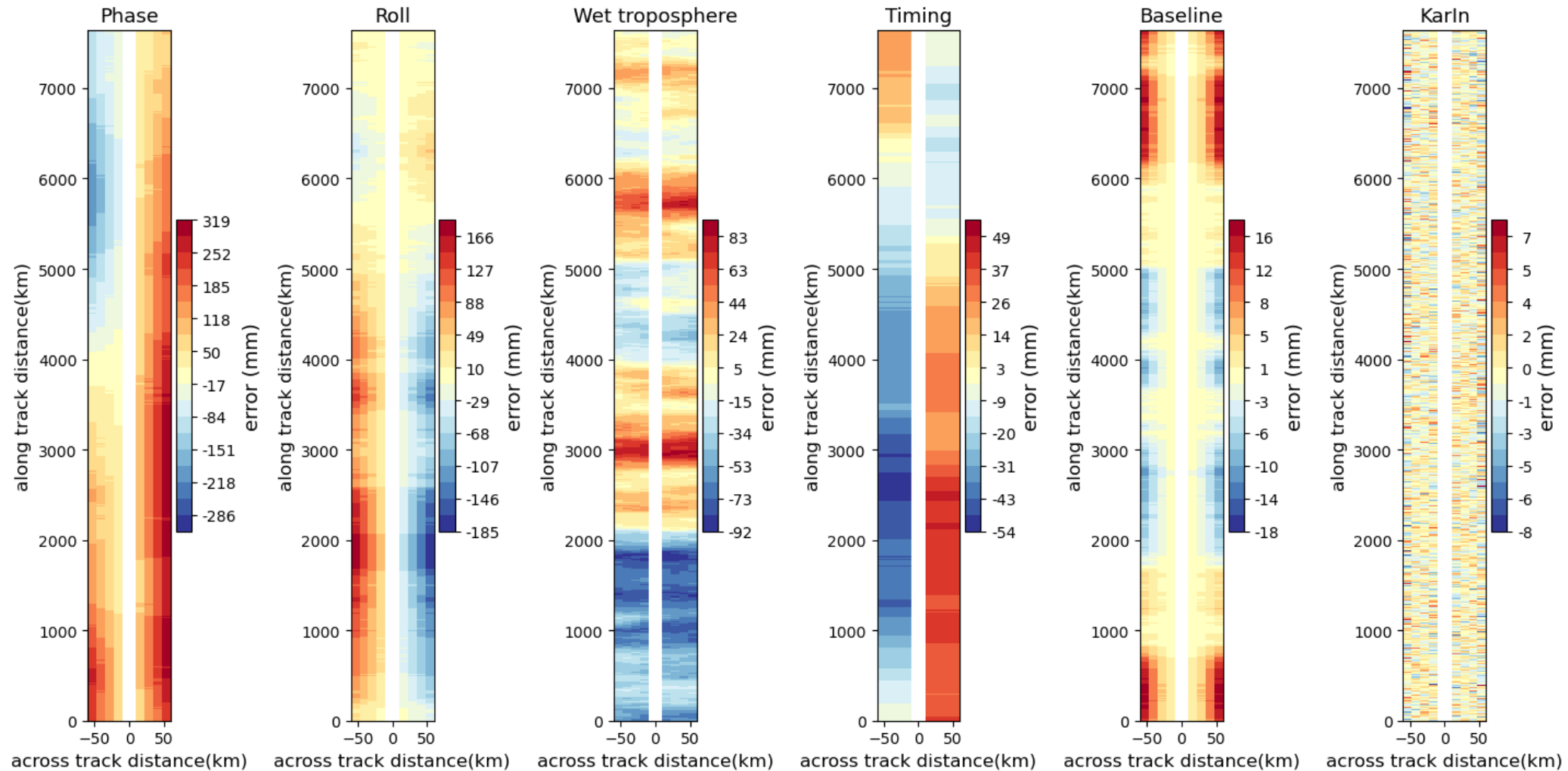
Application: wide swath altimetry

A simulator of SWOT data has been developed by JPL and CNES.

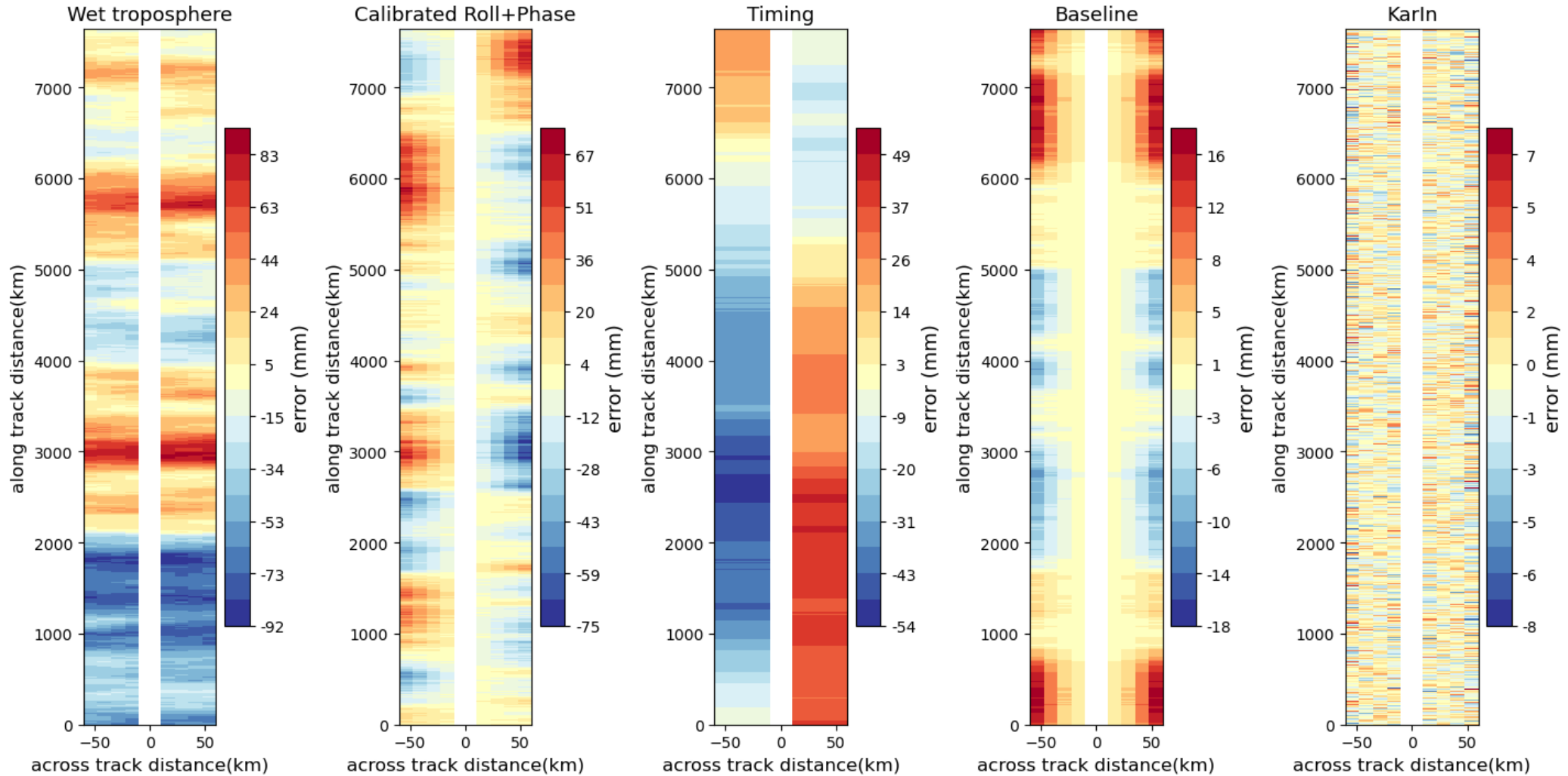




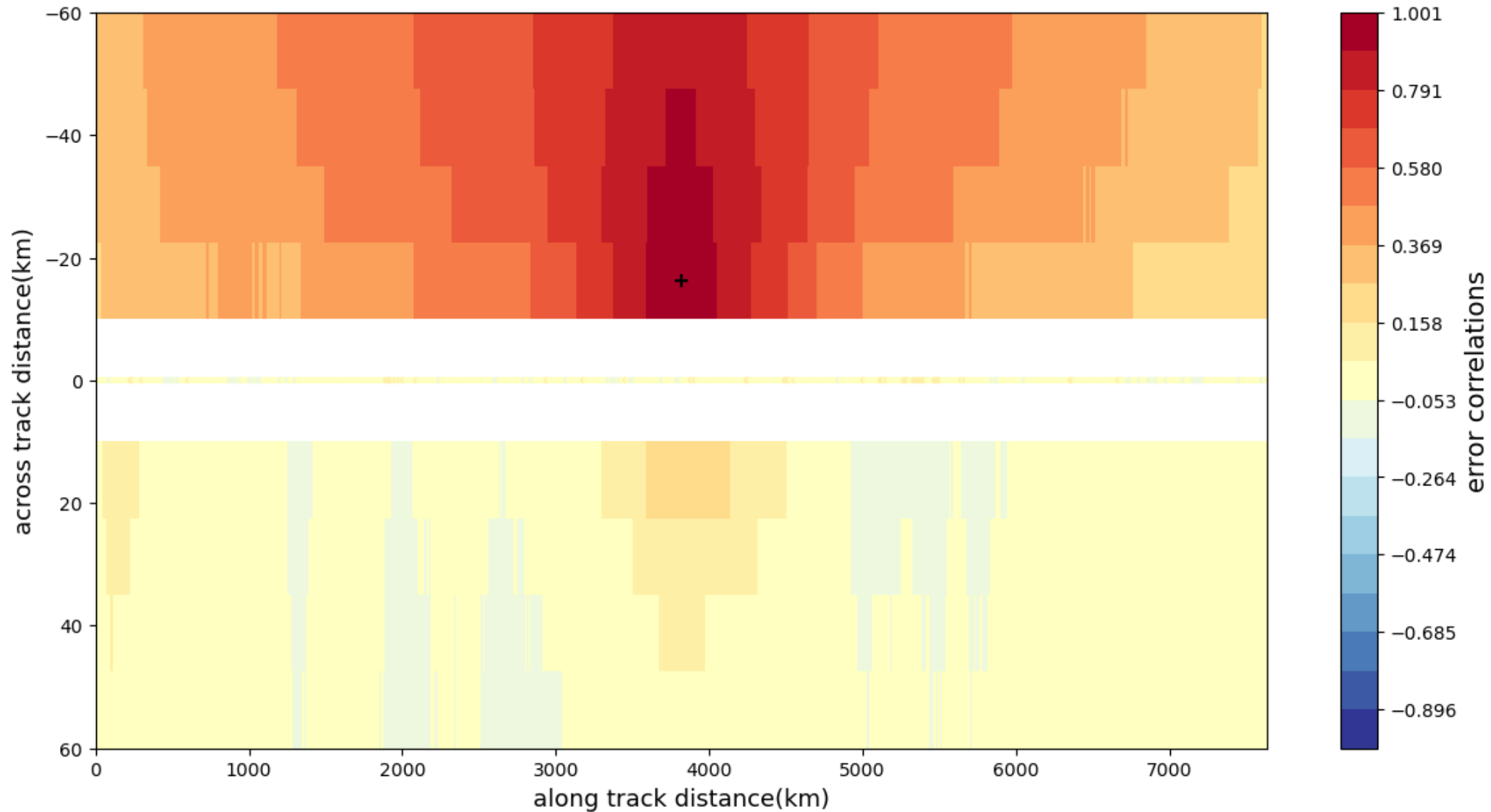
Uncalibrated error sample



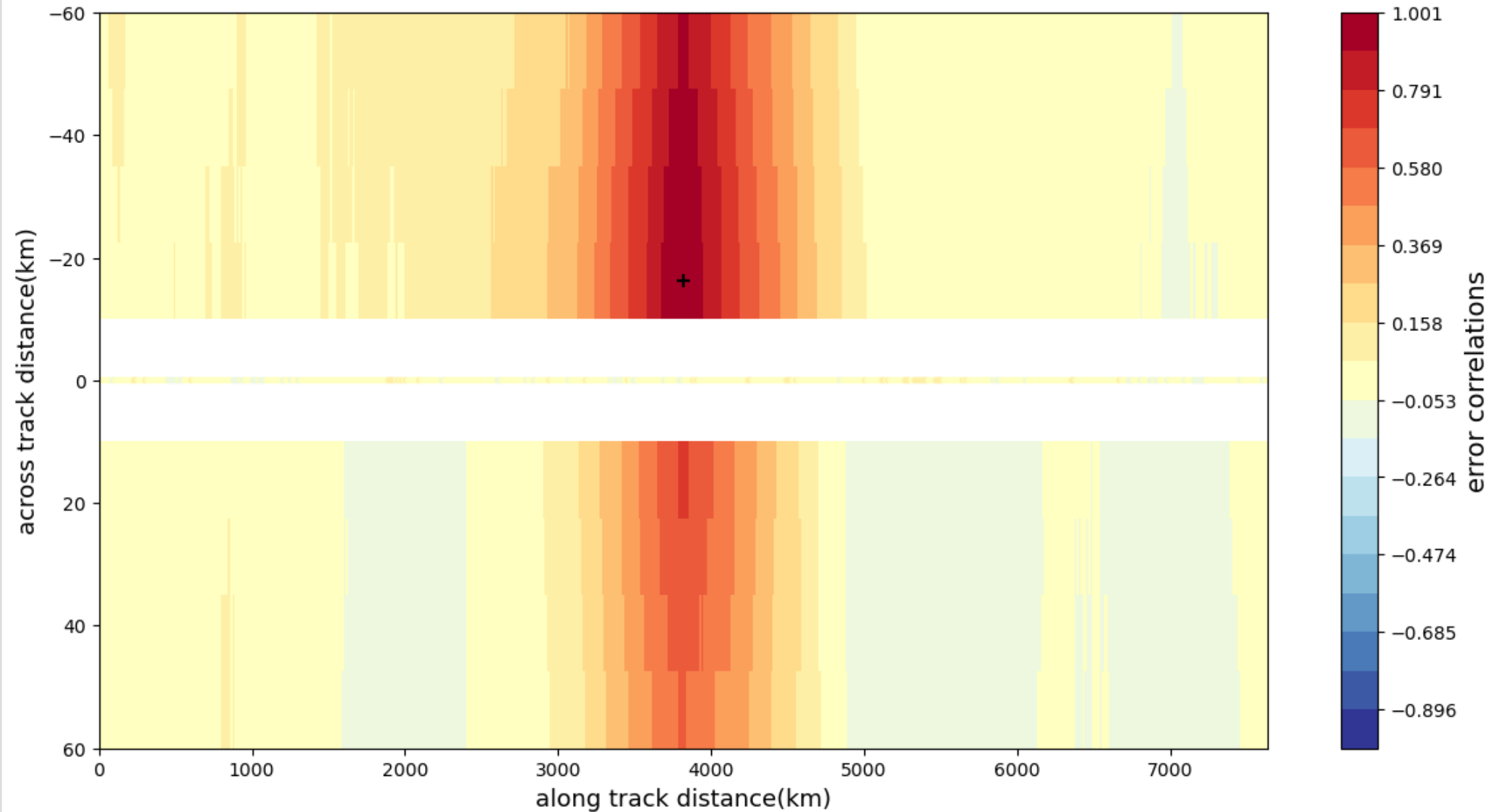
Calibrated error sample



Uncalibrated error correlations

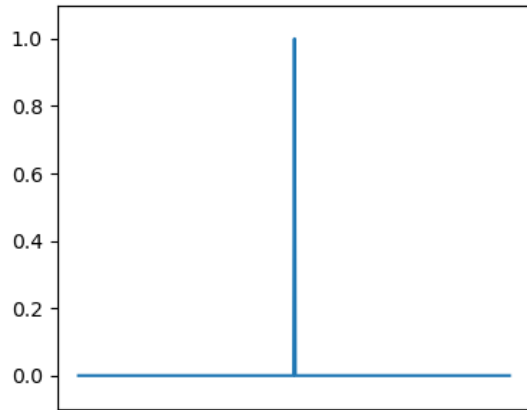


Calibrated error correlations

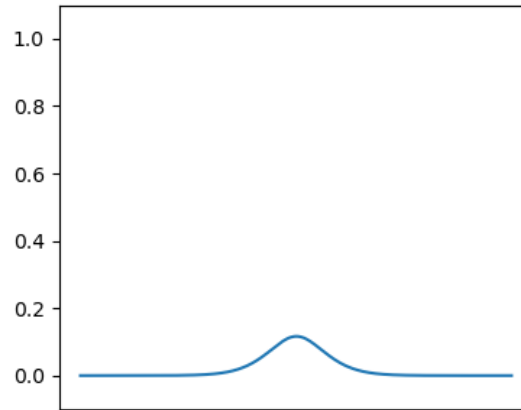


Diffusion-modelled correlation operators

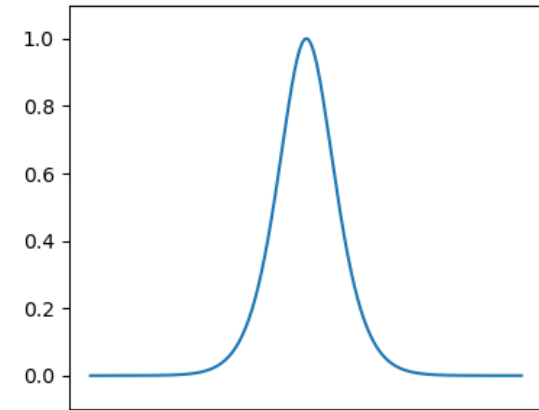
Diffusion operators can be used to create inexpensive, flexible models of R



input vector



smoothed vector



output vector



implicit diffusion



normalization

Both steps are **easy to invert**, and the **inverse operator of R** is even less expensive than the direct operator

Diffusion operators for correlation modelling

Diffusion-modelled correlation operators can be represented by a **sequence of operators**:

$$C = \Gamma \times \underbrace{A^{-1} \times \dots \times A^{-1}}_{M/2} \times W^{-1} \times \underbrace{A^{-T} \dots \times A^{-T}}_{M/2} \times \Gamma$$

Normalization
(diagonal)

Mass matrix
(diagonal)

$I - W^{-1}K$ — Stiffness matrix
(sparse)

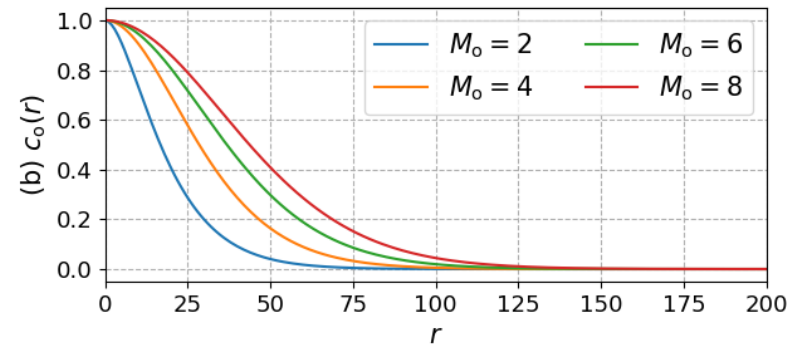
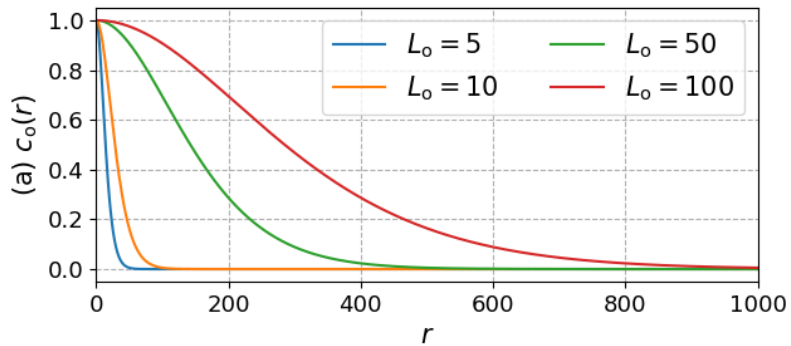
In **variational data assimilation**, we only need the **inverse operator**, which is **inexpensive, scalable and compatible with unstructured data**:

$$C^{-1} = \Gamma^{-1} \times \underbrace{A \times \dots \times A}_{M/2} \times W \times \underbrace{A^T \dots \times A^T}_{M/2} \times \Gamma^{-1}$$

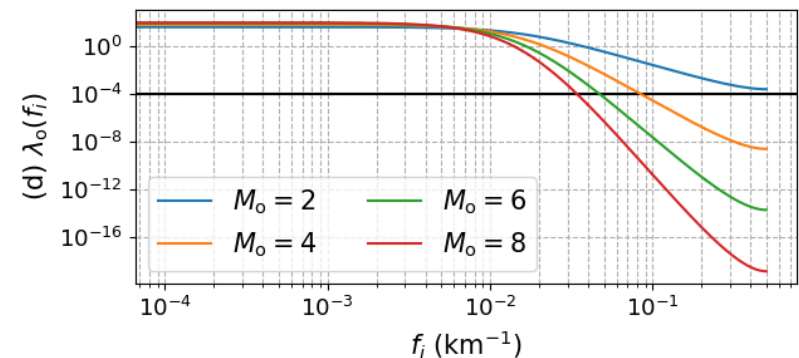
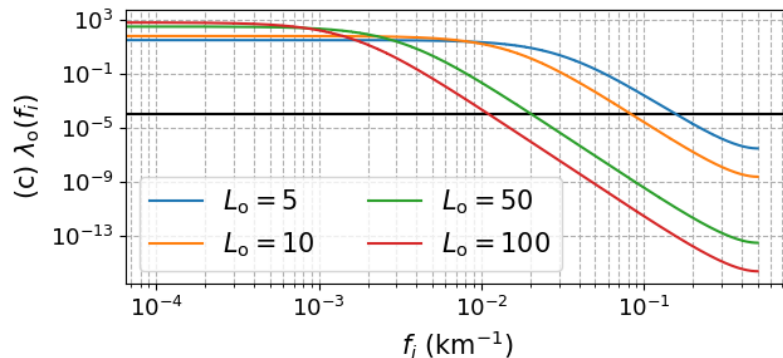


It can be interpreted as a **parametric model** that represents correlation functions from the **Matérn class**.
It depends on a **length-scale parameter L** and an **integer (smoothness) parameter M**

Correlation functions

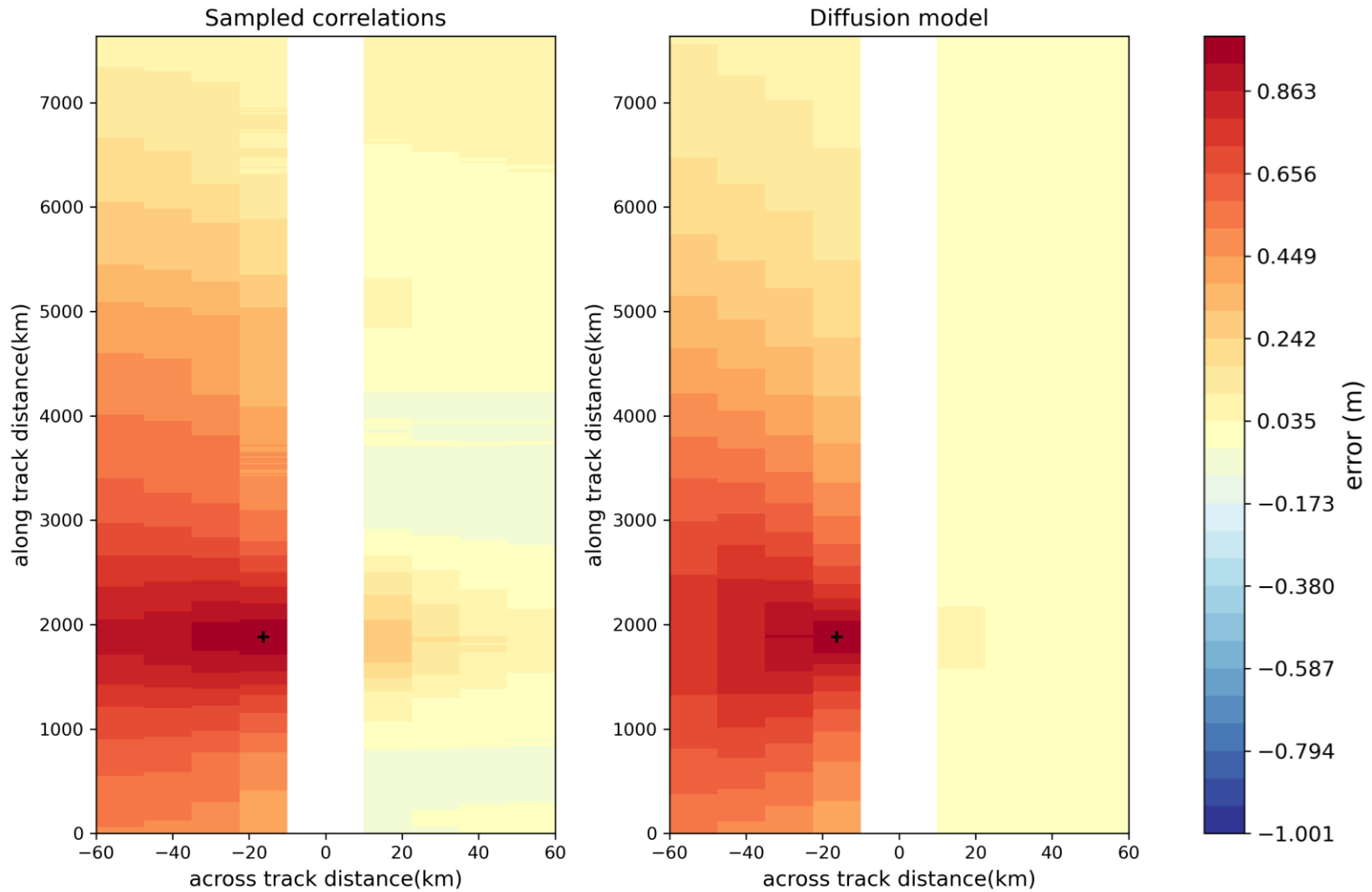


Power spectra

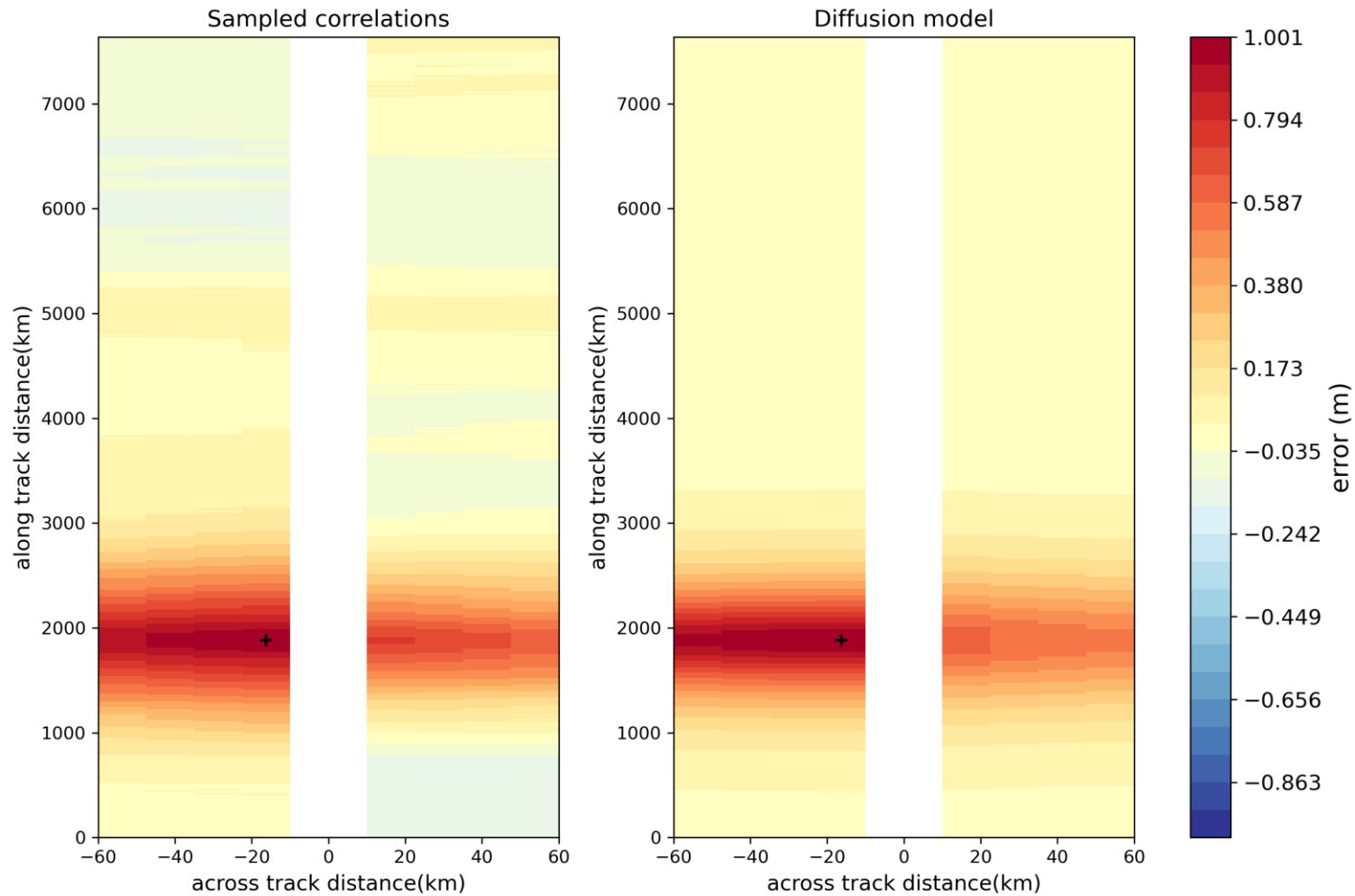


In practice, L can be **spatially variable** and **direction dependent**

Uncalibrated error correlation diffusion model

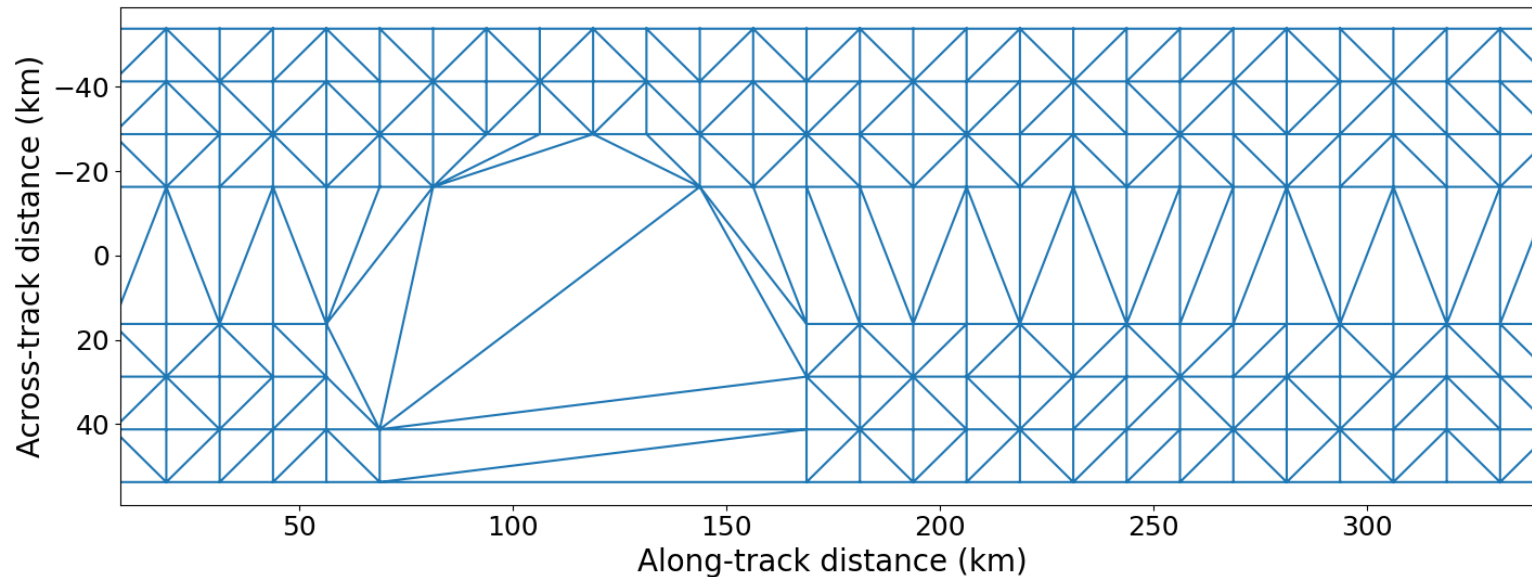
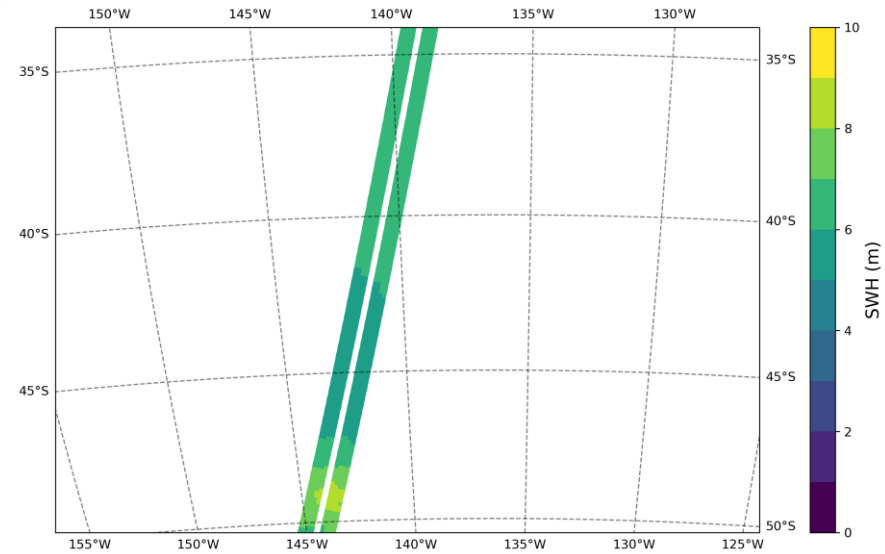


Calibrated error correlation diffusion model



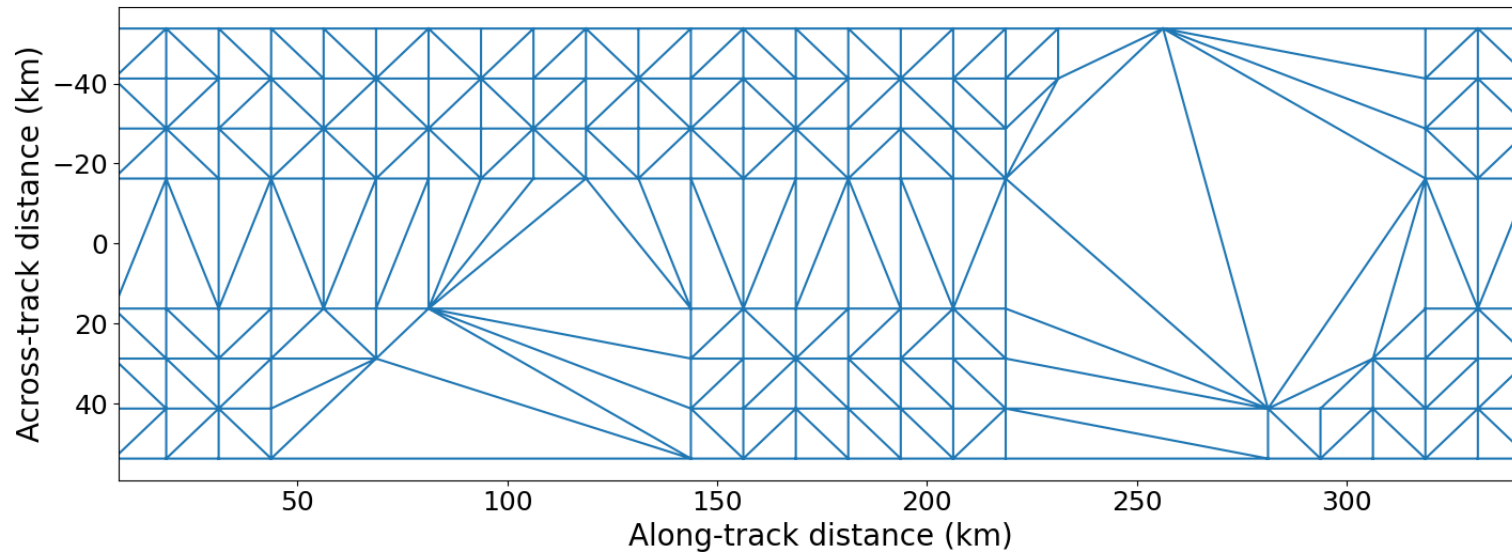
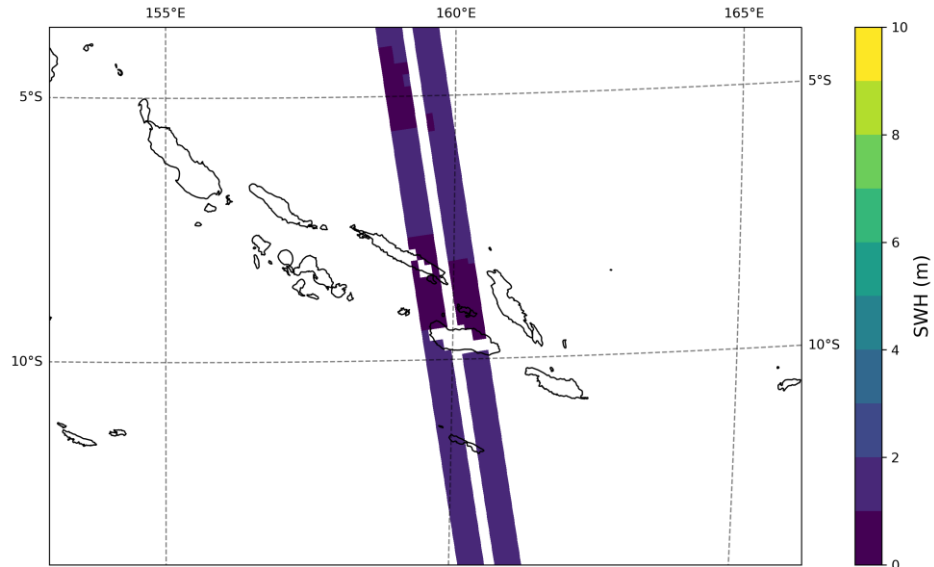
Diffusion on an unstructured mesh

Observations with a large SWH might be removed by the quality control.

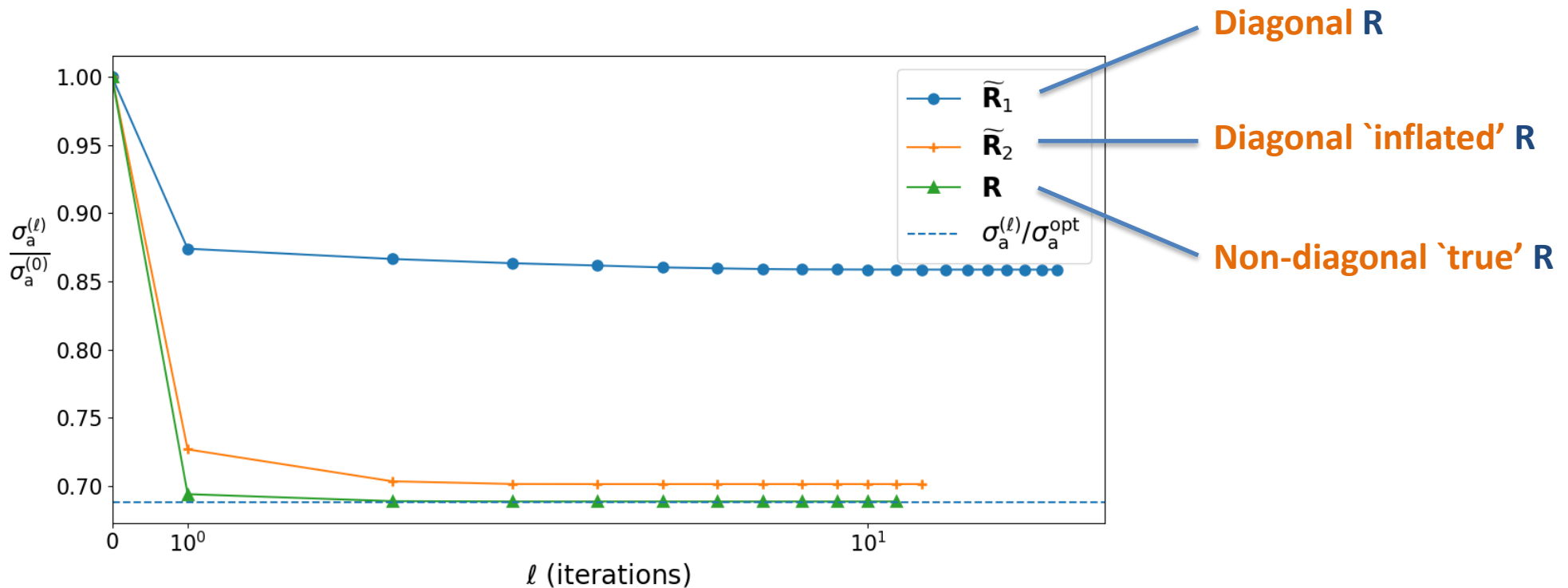


Diffusion on an unstructured mesh

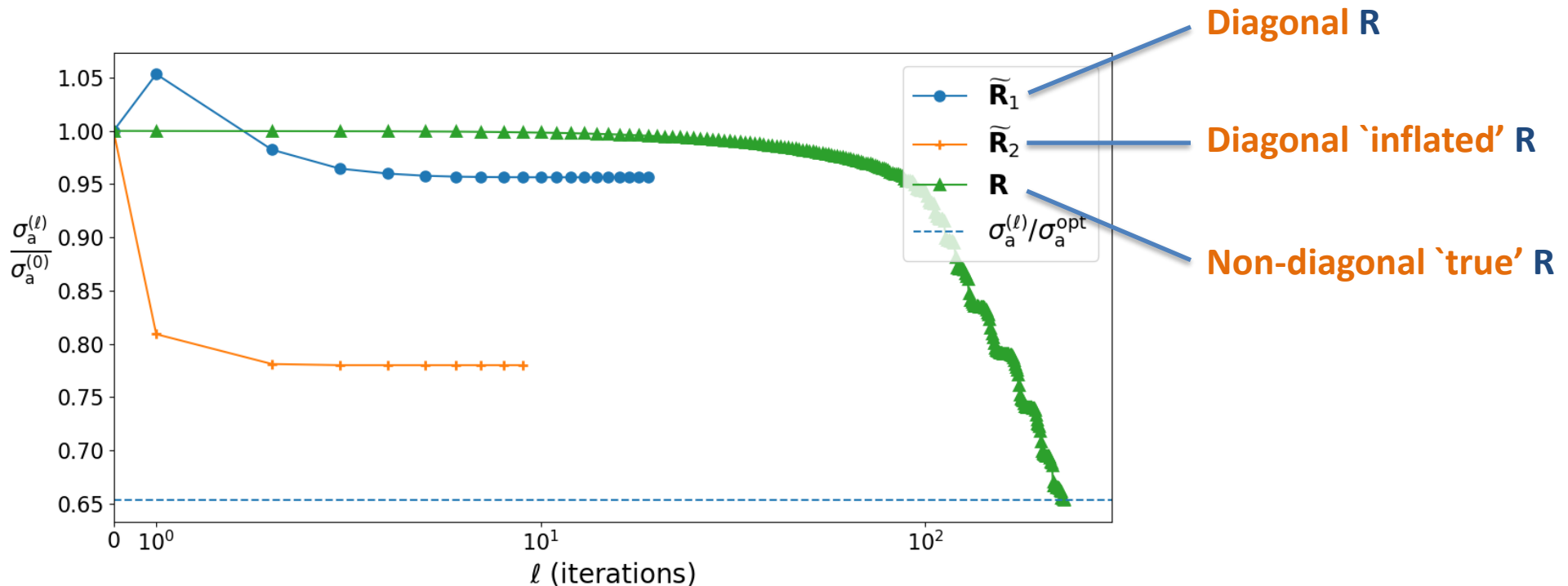
Islands create gaps through which there should be correlations.



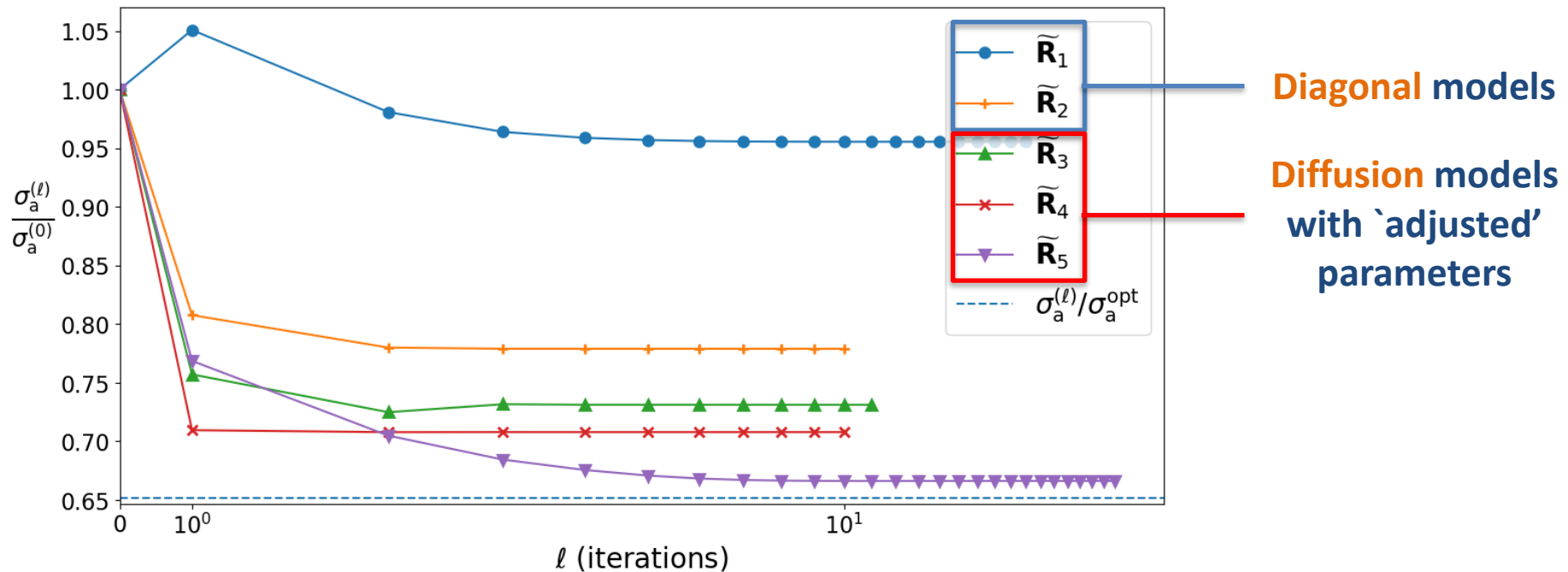
- How will a non-diagonal R impact the convergence of iterative solvers in variational DA ?
- Can a degradation in conditioning outweigh the improvement brought by a more accurate observation error model ?
- Can we adjust the parameters of the observation error correlation model such that the convergence rate of the solvers is improved while staying close to diagnosed correlations ?



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Summary:

- The **assumption of uncorrelated observation error is not realistic** for many observation types, including the soon-to-be-launched SWOT mission.
- **Diffusion operators** can be used to model a **correlation operator, its inverse or its square root**, and can be applied at a low computational cost with unstructured data.
- Using a non-diagonal R in variational DA has a substantial impact on **the convergence of iterative solvers**. If the parameters of the correlation model are chosen wisely, this impact does not have to be negative, and can even **speed up the convergence**.

Perspectives:

- Implementation in the **NEMOVAR** ocean DA system.
- Use this method to account for **correlated errors in altimeter data (nadir as well as wide-swath)**.

Diffusion operators for data assimilation:

Mirouze, I., & Weaver, A. T. (2010). Representation of correlation functions in variational assimilation using an implicit diffusion operator. *Q. J. R. Meteorol. Soc.*, 136(651), 1421–1443. doi: 10.1002/qj.643

Extension to unstructured data:

Guillet, O., Weaver, A. T., Vasseur, X., Michel, Y., Gratton, S., & Gürol, S. (2019). Modelling spatially correlated observation errors in variational data assimilation using a diffusion operator on an unstructured mesh. *Q. J. R. Meteorol. Soc.*, 145(722), 1947–1967. doi: 10.1002/qj.3537

Convergence of the B-PCG:

Goux, O., Gürol, S., Weaver, A. T., Diouane, Y., Guillet, O. (2022) Impact of correlated observation errors on the convergence of the conjugate gradient algorithm in variational data assimilation. *Submitted to Numerical Linear Algebra with Applications.*